

# J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

FYJC TERMINAL TEST - 02

DURATION - 2 HR

MARKS - 50

SOLUTION SET

Q1. (A) Attempt ANY THREE OF THE FOLLOWING

(09)

Q-1A

01.  $kx + 3y + 4 = 0$ ;  $x + ky + 3 = 0$  ;  
 $3x + 4y + 5 = 0$  are consistent . Find k

$$kx + 3y = -4$$

$$x + ky = -3$$

$$3x + 4y = -5 \quad \text{are consistent}$$

$$\begin{vmatrix} k & 3 & -4 \\ 1 & k & -3 \\ 3 & 4 & -5 \end{vmatrix} = 0$$

taking - common from C<sub>1</sub>

$$\begin{vmatrix} + & - & + \\ k & 3 & 4 \\ 1 & k & 3 \\ 3 & 4 & 5 \end{vmatrix} = 0$$

$$k(5k - 12) - 3(5 - 9) + 4(4 - 3k) = 0$$

$$5k^2 - 12k - 3(-4) + 16 - 12k = 0$$

$$5k^2 - 12k + 12 + 16 - 12k = 0$$

$$5k^2 - 24k + 28 = 0$$

$$5k^2 - 10k - 14k + 28 = 0$$

$$5k(k - 2) - 14(k - 2) = 0$$

$$k = 14/5, \quad k = 2$$

02. Find two numbers whose sum is 100 and the ratio of whose AM to GM is 5 : 4

$$a + b = 100 \quad \dots\dots (1)$$

$$\frac{\text{A.M.}}{\text{G.M.}} = \frac{5}{4}$$

$$\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{5}{4}$$

$$\frac{50}{\sqrt{ab}} = \frac{5}{4} \quad \dots\dots \text{from 1}$$

$$\sqrt{ab} = 40$$

$$ab = 1600$$

$$a(100 - a) = 1600$$

$$100a - a^2 = 1600$$

$$a^2 - 100a - 1600 = 0$$

$$(a - 80)(a - 20) = 0$$

$$a = 80; \quad a = 20$$

$$b = 20; \quad b = 80$$

two numbers are : 20, 80

03.  $\sqrt{3} + 3 + 3\sqrt{3} + \dots = 39 + 13\sqrt{3}$  .

Find n

$$a = \sqrt{3}; \quad r = \sqrt{3}; \quad S_n = 39 + 13\sqrt{3}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$39 + 13\sqrt{3} = \frac{\sqrt{3}(\sqrt{3}^n - 1)}{\sqrt{3} - 1}$$

$$13(3 + \sqrt{3}) = \frac{\sqrt{3}(\sqrt{3}^n - 1)}{\sqrt{3} - 1}$$

$$13\sqrt{3}(\sqrt{3} + 1) = \frac{\sqrt{3}(\sqrt{3}^n - 1)}{\sqrt{3} - 1}$$

$$13(\sqrt{3} + 1) = \frac{\sqrt{3}^n - 1}{\sqrt{3} - 1}$$

$$13(\sqrt{3} + 1)(\sqrt{3} - 1) = \sqrt{3}^n - 1$$

$$13(3 - 1) = \sqrt{3}^n - 1$$

$$26 = \sqrt{3}^n - 1$$

$$27 = \sqrt{3}^n$$

$$3^3 = \sqrt{3}^n$$

$$\sqrt{3}^6 = \sqrt{3}^n$$

$$n = 6$$

04. sum is 60 and the ratio of the product of the second and third term to the product of the first and the fourth term is 3 : 2

SOLUTION

let 4 nos in AP :  $a - 3d, a - d, a + d, a + 3d$

$$\text{sum} = 60$$

$$4a = 60 \quad \therefore a = 15$$

ratio of the product of the second and third term to the product of the first and the fourth term is 3 : 2

$$\frac{(a - d)(a + d)}{(a - 3d)(a + 3d)} = \frac{3}{2}$$

$$\frac{a^2 - d^2}{a^2 - 9d^2} = \frac{3}{2}$$

$$2a^2 - 2d^2 = 3a^2 - 27d^2$$

$$25d^2 = a^2$$

$$25d^2 = 225$$

$$d^2 = 9 \quad \therefore d = \pm 3$$

$$a = 15, d = 3$$

nos are :  $a - 3d, a - d, a + d, a + 3d$

$$15 - 9, 15 - 3, 15 + 3, 15 + 9$$

$$6, 12, 18, 24$$

$$a = 15, d = -3$$

nos are :  $a - 3d, a - d, a + d, a + 3d$

$$15 + 9, 15 + 3, 15 - 3, 15 - 9$$

$$24, 18, 12, 6$$

# Q-1B

01. 
$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$$

taking 'a', 'b' & 'c' common from C<sub>1</sub>, C<sub>2</sub> & C<sub>3</sub> respectively

$$= abc \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

C<sub>1</sub> - C<sub>2</sub>, C<sub>2</sub> - C<sub>3</sub>

$$= abc \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \end{vmatrix}$$

Taking (a - b) common from C<sub>1</sub> & (b - c) common from C<sub>2</sub>

$$= abc(a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a+b & b+c & c^2 \end{vmatrix}$$

C<sub>1</sub> - C<sub>2</sub>

$$= abc(a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ a-c & b+c & c^2 \end{vmatrix}$$

Taking (a - c) common from C<sub>1</sub>

$$= abc(a-b)(b-c)(a-c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ 1 & b+c & c^2 \end{vmatrix}$$

Expanding the determinant

$$= abc(a-b)(b-c)(a-c) [1(0-1)]$$

$$= abc(a-b)(b-c)(a-c) (-1)$$

$$= abc(a-b)(b-c)(c-a)$$

02. 
$$\begin{vmatrix} x+y & y+z & z+x \\ z+x & x+y & y+z \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix}$$

LHS

C<sub>1</sub> - C<sub>2</sub>

$$= \begin{vmatrix} x-z & y+z & z+x \\ z-y & x+y & y+z \\ y-x & z+x & x+y \end{vmatrix}$$

C<sub>1</sub> + C<sub>3</sub>

$$= \begin{vmatrix} 2x & y+z & z+x \\ 2z & x+y & y+z \\ 2y & z+x & x+y \end{vmatrix}$$

Taking '2' common from C<sub>1</sub>

$$= 2 \begin{vmatrix} x & y+z & z+x \\ z & x+y & y+z \\ y & z+x & x+y \end{vmatrix}$$

C<sub>3</sub> - C<sub>1</sub>

$$= 2 \begin{vmatrix} x & y+z & z \\ z & x+y & y \\ y & z+x & x \end{vmatrix}$$

C<sub>2</sub> - C<sub>3</sub>

$$= 2 \begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix} = \text{RHS}$$

Q2. (A) Attempt ANY TWO OF THE FOLLOWING 01. Find equation of the line passing through centroid of triangle ABC whose vertices are A(1,6) , B(-2,9) and C(-2 , 3) such that sum of its intercepts on the coordinate axes is 6 .

$$G \equiv \left( \frac{1-2-2}{3}, \frac{6+9+3}{3} \right) = (-1, 6)$$

$a + b = 6$  ..... given

$b = 6 - a$  .... (1)

let the equation of the line be

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{a} + \frac{y}{6-a} = 1$$

since the line passes through (-1,6) , it must satisfy the equation

$$\frac{-1}{a} + \frac{6}{6-a} = 1$$

$$-6 + a + 6a = a(6 - a)$$

$$-6 + 7a = 6a - a^2$$

$$a^2 + a - 6 = 0$$

$$a^2 + 3a - 2a - 6 = 0$$

$$(a + 3)(a - 2) = 0$$

$a = -3$

$b = 6 - a$

$= 6 + 3$

$= 9$

Equation

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{-3} + \frac{y}{9} = 1$$

$$9x - 3y = -27$$

$$3x - y + 9 = 0$$

$a = 2$

$b = 6 - a$

$= 6 - 2$

$= 4$

Equation

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{2} + \frac{y}{4} = 1$$

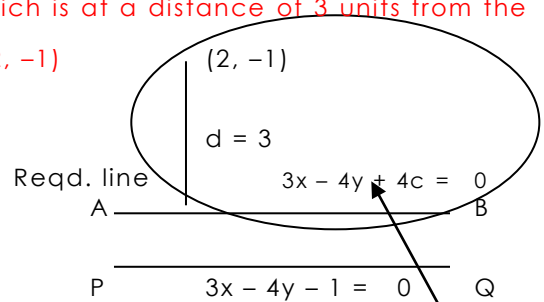
$$\frac{2x + y}{4} = 1$$

$$2x + y = 4$$

Q-2A

02.

Find equation of line parallel to  $3x - 4y - 1 = 0$  and which is at a distance of 3 units from the point (2, -1)



STEP 1 : PQ

$$3x - 4y - 1 = 0 \quad m = \frac{-a}{b} = \frac{-3}{-4} = \frac{3}{4}$$

STEP 2 : AB

$$m_{AB} = \frac{3}{4} \text{ (AB//PQ)}$$

equation of AB :  $y = mx + c$

$$y = \frac{3x}{4} + c$$

$$y = \frac{3x + 4c}{4}$$

$$4y = 3x + 4c$$

$$3x - 4y + 4c = 0$$

STEP 3 :

$d = 3$

$$\left| \frac{3(2) - 4(-1) + 4c}{\sqrt{3^2 + 4^2}} \right| = 3$$

$$\left| \frac{6 + 4 + 4c}{5} \right| = 3$$

$$\left| \frac{10 + 4c}{5} \right| = 3$$

$$\frac{10 + 4c}{5} = \pm 3$$

$$10 + 4c = \pm 15$$

$$10 + 4c = 15$$

$$4c = 5$$

Equation of AB

$$3x - 4y + 5 = 0$$

$$10 + 4c = -15$$

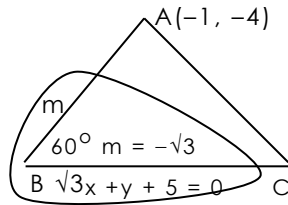
$$4c = -25$$

Equation of AB

$$3x - 4y - 25 = 0$$

**03.**

find equation of the lines which pass through point  $(-1, -4)$  and inclined at an angle of  $60^\circ$  to the line  $\sqrt{3}x + y + 5 = 0$



STEP 1 :

$$\sqrt{3}x + y + 5 = 0$$

$$m = \frac{-a}{b} = \frac{-\sqrt{3}}{1} = -\sqrt{3}$$

STEP 2 :

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right|$$

$$\tan 60 = \left| \frac{m + \sqrt{3}}{1 + m(-\sqrt{3})} \right|$$

$$\sqrt{3} = \left| \frac{m + \sqrt{3}}{1 - \sqrt{3}m} \right|$$

$\frac{m + \sqrt{3}}{1 - \sqrt{3}m} = \sqrt{3}$	$\frac{m + \sqrt{3}}{1 - \sqrt{3}m} = -\sqrt{3}$
$m + \sqrt{3} = \sqrt{3} - 3m$	$m + \sqrt{3} = -\sqrt{3} + 3m$
$m + 3m = \sqrt{3} - \sqrt{3}$	$m - 3m = -\sqrt{3} - \sqrt{3}$
$4m = 0$	$-2m = -2\sqrt{3}$
$m = 0$	$m = \sqrt{3}$

STEP 3

Equation of AB :  $m = 0$  ,  $A(-1, -4)$

$$y - y_1 = m(x - x_1)$$

$$y + 4 = 0(x + 1)$$

$$y + 4 = 0$$

Equation of AC :  $m = \sqrt{3}$  ,  $A(-1, -4)$

$$y - y_1 = m(x - x_1)$$

$$y + 4 = \sqrt{3}(x + 1)$$

$$y + 4 = \sqrt{3}x + \sqrt{3}$$

$$\sqrt{3}x - y + \sqrt{3} - 4 = 0$$

**B) Attempt ANY TWO OF THE FOLLOWING**

(06)

**Q-2B**

01.

$$8\sin x - \cos x = 4$$

$$8\sin x - 4 = \cos x$$

$$(8\sin x - 4)^2 = \cos^2 x$$

$$64 \sin^2 x - 64 \sin x + 16 = 1 - \sin^2 x$$

$$65 \sin^2 x - 64 \sin x + 15 = 0$$

$$65 \sin^2 x - 39 \sin x - 25 \sin x + 15 = 0$$

$$13\sin x (5\sin x - 3) - 5(5\sin x - 3) = 0$$

$$(13\sin x - 5)(5\sin x - 3) = 0$$

$$\sin x = \frac{5}{13} \quad \text{OR} \quad \sin x = \frac{3}{5}$$

02

$$\frac{1}{\tan 3\theta - \tan \theta} + \frac{1}{\cot \theta - \cot 3\theta} = \cot 2\theta$$

LHS

$$= \frac{1}{\tan 3\theta - \tan \theta} + \frac{1}{\cot \theta - \cot 3\theta}$$

$$= \frac{1}{\tan 3\theta - \tan \theta} + \frac{1}{\frac{1}{\tan \theta} - \frac{1}{\tan 3\theta}}$$

$$= \frac{1}{\tan 3\theta - \tan \theta} + \frac{\tan 3\theta \cdot \tan \theta}{\tan 3\theta - \tan \theta}$$

$$= \frac{1 + \tan 3\theta \cdot \tan \theta}{\tan 3\theta - \tan \theta}$$

$$= \frac{1}{\frac{\tan 3\theta - \tan \theta}{1 + \tan 3\theta \cdot \tan \theta}}$$

$$= \frac{1}{\tan (3\theta - \theta)}$$

$$= \frac{1}{\tan 2\theta}$$

$$= \cot 2\theta = \text{RHS}$$

$$03. \frac{\cos 15^\circ + \sin 15^\circ}{\cos 15^\circ - \sin 15^\circ} = \sqrt{3}$$

$$\tan 60^\circ = \tan (45 + 15)$$

$$\sqrt{3} = \frac{\tan 45^\circ + \tan 15^\circ}{1 - \tan 45^\circ \cdot \tan 15^\circ}$$

$$= \frac{1 + \tan 15^\circ}{1 - \tan 15^\circ}$$

$$= \frac{1 + \frac{\sin 15^\circ}{\cos 15^\circ}}{1 - \frac{\sin 15^\circ}{\cos 15^\circ}}$$

$$\sqrt{3} = \frac{\cos 15^\circ - \sin 15^\circ}{\cos 15^\circ + \sin 15^\circ}$$

01. How many different arrangements can be made using all letters of the word 'ENGLISH'

'ENGLISH' : 7 – LETTER WORD

VOWELS :- E , I ; CONSONANTS :- N , G , L , S , H

Q-3A

a) how many of them begin with N and end with L

1<sup>ST</sup> place can be filled by letter 'N' in 1 way

Last place can be filled by letter 'L' in 1 way

Having done that ,

Remaining 5 places can be filled by remaining 5 letters in  ${}^5P_5 = 5!$  ways

By fundamental principle of Multiplication

$$\text{Total arrangements} = 1 \times 1 \times 5! = 120$$

b) in how many of them do vowel occupy even places

2 vowels can be arranged into any 2 out of the 3 even places in  ${}^3P_2$  ways

Having done that

Remaining 5 places can be filled by remaining 5 consonants in  ${}^5P_5 = 5!$  ways

By fundamental principle of Multiplication

$$\text{Total arrangements} = {}^3P_2 \times 5! = 6 \times 120 = 720$$

c) in how many of these vowels are always together

consider 2 vowels as 1 set .

1 set of 2 vowels & 5 consonants can be arranged in  ${}^6P_6 = 6!$  Ways

Having done that ;

The 2 vowels can then be arranged amongst themselves in  ${}^2P_2 = 2!$  ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 6! \times 2! \\ &= 720 \times 2 \\ &= 1440 \end{aligned}$$

02. A family of 3 sisters and 5 brothers is to be arranged for a photograph in one row . In how many ways can they be seated if

- a) all the sisters sit together      b) no two sisters sit together

a) all the sisters sit together

Consider 3 sisters as 1 set

1 set of 3 sisters & 5 Brothers can be arranged in  ${}^6P_6 = 6!$  ways

Having done that ;

3 sisters can then be arranged among themselves in  ${}^3P_3 = 3!$  ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 6! \times 3! \\ &= 720 \times 6 = 4320 \end{aligned}$$

b) no two sisters sit together

\* B \* B \* B \* B \* B \*

3 sisters can be arranged into any 3 of the 6 positions marked '\*' in  ${}^6P_3$  ways

Having done that ;

5 Brothers can then be arranged in  ${}^5P_5 = 5!$  ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= {}^6P_3 \times 5! \\ &= 120 \times 120 = 14400 \end{aligned}$$



(B) Attempt ANY THREE OF THE FOLLOWING

(09)

01. From the following data, how many values of the given data are more than 28

CI	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No. of worker:	7	11	20	8	4

Q-3B

SOLUTION :

CI	f	cf
0 - 10	7	7
10 - 20	11	18
20 - 30	20	38 ←
30 - 40	8	46
40 - 50	4	50

let nth observation have the value '28'

Its in the class 20 - 30

$$28 = 20 + \frac{n - 18}{20} (30 - 20)$$

$$8 = \frac{n - 18}{20} \cdot (10)$$

$$8 = \frac{n - 18}{2}$$

$$16 = n - 18 \quad \therefore n = 34$$

34<sup>th</sup> observation has the value 28

$$\therefore \text{no. of observations having value more than 28} = 50 - 34 = 16$$

02.

Marks	no of students
more than 0	50
more than 10	46
more than 20	40
more than 30	20
more than 40	10
more than 50	3

CI	f	cf
0 - 10	4	4
10 - 20	6	10
20 - 30	20	30
30 - 40	10	40
40 - 50	7	47
50 - 60	3	50

$$d_7 = 4 \frac{N}{10} = \frac{4(50)}{10} = 20$$

$$D_4 = L_1 + \frac{d_4 - c}{f} (L_2 - L_1)$$

$$= 20 + \frac{20 - 10}{20} (30 - 20)$$

$$= 20 + \frac{10}{20} (10)$$

$$= 20 + 5$$

$$= 25 \text{ marks}$$

03.  $\frac{\log_2 a}{4} = \frac{\log_2 b}{6} = \frac{\log_2 c}{3k}$  and  $a^3 \cdot b^2 \cdot c = 1$ . Find k

SOLUTION

$$\frac{\log_2 a}{4} = \frac{\log_2 b}{6} = \frac{\log_2 c}{3k} = m$$

$$\log_2 a = 4m \quad \therefore a = 2^{4m}$$

$$\log_2 b = 6m \quad \therefore b = 2^{6m}$$

$$\log_2 c = 3km \quad \therefore c = 2^{3km}$$

$$a^3 \cdot b^2 \cdot c = 1$$

$$2^{4m \cdot 3} \cdot 2^{6m \cdot 2} \cdot 2^{3km} = 1$$

$$2^{12m} \cdot 2^{12m} \cdot 2^{3km} = 1$$

$$2^{12m + 12m + 3km} = 1$$

$$2^{24m + 3km} = 2^0$$

EQUATING THE POWERS

$$24m + 3km = 0$$

$$3km = -24m$$

$$3k = -24$$

$$k = -8$$

04. if  $a^x = b^y = c^z$  and  $b^2 = ac$  then prove that  $y = \frac{2xz}{x+z}$

SOLUTION

$$a^x = b^y = c^z$$

$$\log a^x = \log b^y = \log c^z$$

$$x \log a = y \log b = z \log c = k$$

$$\log a = \frac{k}{x}, \quad \log b = \frac{k}{y}; \quad \log c = \frac{k}{z}$$

NOW

$$b^2 = ac$$

$$\log b^2 = \log ac$$

$$2 \log b = \log a + \log c$$

$$2 \frac{k}{y} = \frac{k}{x} + \frac{k}{z}$$

$$\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$$\frac{2}{y} = \frac{x+z}{xz}$$

$$y = \frac{2xz}{x+z} \quad \dots\dots \text{PROVED}$$

Q-4A

01.

the mean and standard deviation of 100 observations were found to be 6 and 2 respectively . If at the time of calculation , one observation was wrongly taken as 17 instead of 7 . Find the correct standard deviation

$$\bar{x} = 6 \text{ \& } \sigma = 2, n = 100$$

$$\text{incorrect } x = 17,$$

$$\text{correct } x = 7$$

STEP 1 : CORRECTION OF MEAN

$$\bar{x} = \frac{\sum x}{n}$$

$$6 = \frac{\sum x}{100}$$

$$\begin{array}{r} \sum x = 600 \\ - \text{incorrect } x = 17 \\ + \text{correct } x = 7 \\ \hline \sum x \text{ correct} = 590 \end{array}$$

$$\bar{x}_{\text{correct}} = \frac{\sum x}{n}$$

$$= \frac{590}{100}$$

$$= 5.9$$

STEP 2 : CORRECTION OF S.D.

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

$$\sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2$$

$$\begin{aligned} \sum x^2 &= n(\sigma^2 + \bar{x}^2) \\ &= 100(2^2 + 6^2) \\ &= 100(4 + 36) \\ &= 4000 \end{aligned}$$

Now

$$\begin{array}{r} \sum x^2 = 4000 \\ - \text{incorrect } x^2 = 289 \\ + \text{correct } x^2 = 49 \\ \hline \sum x^2 \text{ correct} = 3760 \end{array}$$

$$\sigma_{\text{correct}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

CORRECT MEAN

$$= \sqrt{\frac{3760}{100} - 5.9^2}$$

$$= \sqrt{37.60 - 34.81}$$

$$= \sqrt{2.79}$$

taking log on both sides

$$\begin{aligned} \log \sigma &= \frac{1}{2}(\log 2.79) \\ &= \frac{1}{2}(0.4456) \\ &= \frac{0.4456}{2} \\ \log \sigma &= 0.2228 \\ \sigma_{\text{correct}} &= \text{AL}(0.2228) \\ &= 1.670 \end{aligned}$$

02.

first 3 moments about 7 calculated from a set of 9 observations are 0.2 ; 19.4 and -41 respectively . Find the mean , variance and the second raw moment of the distribution

$$A = 7 ,$$

$$\mu_1(a) = 0.2 , \mu_2(a) = 19.4, \mu_3(a) = -41$$

$$\mu_1(a) = \bar{x} - A$$

$$0.2 = \bar{x} - 7$$

$$\bar{x} = 7.2 \quad (\text{mean})$$

$$\mu_2 = \mu_2(a) - \mu_1(a)^2$$

$$= 19.4 - 0.2^2$$

$$= 19.4 - 0.04$$

$$= 19.36$$

$$\text{variance } (\sigma^2) = \mu_2 = 19.36$$

Now

$$\sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2$$

$$19.36 = \frac{\sum x^2}{n} - 7.2^2$$

$$19.36 = \frac{\sum x^2}{n} - 51.84$$

$$\frac{\sum x^2}{n} = 19.36 + 51.84 = 71.2$$

SECOND RAW MOMENT

$$\mu_2' = \frac{\sum x^2}{n} = 71.2$$

03.

the mean & variance of a distribution are 50 and 400 respectively . Find the mode and the median if  $SK_p = -0.4$

$$\text{Mean} = 50 , \sigma^2 = 400 , SK_p = -0.4$$

**STEP 1 : MODE**

$$SK_p = \frac{\text{Mean} - \text{Mode}}{\sigma}$$

$$-0.4 = \frac{50 - \text{Mode}}{20}$$

$$-8 = 50 - \text{Mode}$$

$$\text{Mode} = 50 + 8$$

$$= 58$$

**STEP 2 : MEDIAN**

$$\text{Mean} - \text{mode} = 3(\text{mean} - \text{median})$$

$$50 - 58 = 3(50 - \text{median})$$

$$-8 = 3(50 - \text{median})$$

$$-2.67 = 50 - \text{median}$$

$$\text{median} = 52.67$$

# Q-4B

01. the first four raw moments are 2, 20, 40, 800 respectively. Find the coefficient of kurtosis  $\gamma_2$

moments about  $A = 0$ ,

$$\mu_1(a) = 2, \mu_2(a) = 20, \mu_3(a) = 40$$

$$\mu_4(a) = 800$$

$$\mu_1 = 0$$

$$\mu_2 = \mu_2(a) - \mu_1(a)^2$$

$$= 20 - 4$$

$$= 16$$

$$\mu_3 = \mu_3(a) - 3\mu_1(a)\mu_2(a) + 2\mu_1(a)^3$$

$$\mu_4 = \mu_4(a) - 4\mu_1(a)\mu_3(a) + 6\mu_1(a)^2\mu_2(a) - 3\mu_1(a)^4$$

$$= 800 - 4(2)(40) + 6(4)(20) - 3(16)$$

$$= 800 - 320 + 480 - 48$$

$$= 912$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{912}{256}$$

$$= 3.5625$$

$$\gamma_2 = \beta_2 - 3 = 0.5625$$

02.

$${}^{n+2}P_4 : {}^{n+3}P_6 = 1 : 14$$

$$\frac{{}^{n+2}P_4}{{}^{n+3}P_6} = \frac{1}{14}$$

$$\frac{(n+2)!}{(n+2-4)!} = \frac{1}{14} \cdot \frac{(n+3)!}{(n+3-6)!}$$

$$\frac{(n+2)!}{(n-2)!} = \frac{1}{14} \cdot \frac{(n+3)!}{(n-3)!}$$

$$\frac{(n+2)!}{(n-2)!} \times \frac{(n-3)!}{(n+3)!} = \frac{1}{14}$$
$$\frac{(n+2)!}{(n+3)!} \times \frac{(n-3)!}{(n-2)!} = \frac{1}{14}$$

$$\frac{(n+2)!}{(n+3)(n+2)!} \times \frac{(n-3)!}{(n-2)(n-3)!} = \frac{1}{14}$$

$$\frac{1}{(n+3)(n-2)} = \frac{1}{14}$$

$$(n+3)(n-2) = 14$$

$$(n+3)(n-2) = 7 \cdot 2$$

On Comparing ;

$$n+3 = 7$$

$$n = 7 - 3 = 4$$

03. How many 5 digit numbers can be formed by using all the digits 2 , 3 , 4 , 0 , 9 . From them how numbers are more than 40000 and divisible by 2 .

ten thousand place must not contain '0' . Hence it can be filled by any one of the remaining 4 digits in  ${}^4P_1$  ways

Having done that ,

The remaining 4 places can be filled by the remaining 4 digits in  ${}^4P_4 = 4!$  Ways

By Fundamental Principle of Multiplication

Total 5 digit numbers formed =  ${}^4P_1 \times 4! = 4 \times 24 = 96$

numbers are more than 40000 and divisible by 2

Case 1 : Numbers between 40000 & 50000

Ten thousand place can be filled by digit '4' in 1 way

Unit place can be filled by any one of the digits 0 & 2 in  ${}^2P_1$  ways

Having done that ,

The remaining 3 places can be filled by the remaining 3 digits in  ${}^3P_3 = 3!$  Ways

By Fundamental Principle of Multiplication

numbers formed =  ${}^2P_1 \times 3! = 2 \times 6 = 12$

Case 2 : Numbers exceeding 50000

Ten thousand place can be filled by digit '9' in 1 way

Unit place can be filled by any one of the digits 0 , 2 & 4 in  ${}^3P_1$  ways

Having done that ,

The remaining 3 places can be filled by the remaining 3 digits in  ${}^3P_3 = 3!$  Ways

By Fundamental Principle of Multiplication

numbers formed =  ${}^3P_1 \times 3! = 3 \times 6 = 18$

By Fundamental Principle of ADDITION

Total numbers formed =  $12 + 18 = 30$

*All The Best for Terminal exams ,  
Ashish Sir @ JKSC*